1.

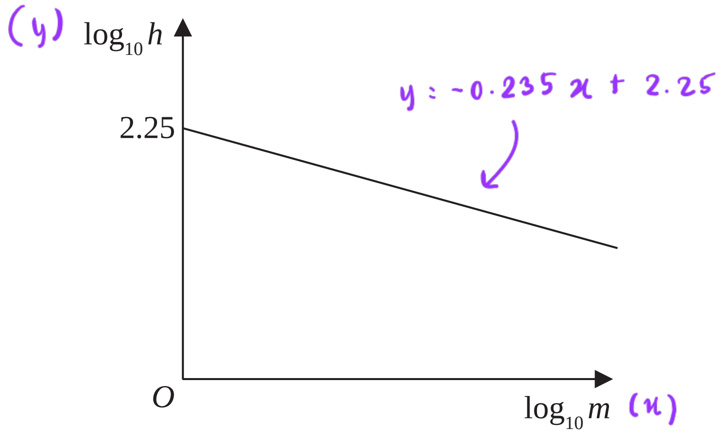


**Figure 2**

The resting heart rate, $h$, of a mammal, measured in beats per minute, is modelled by the equation

$$h = pm^q$$

where $p$ and $q$ are constants and $m$ is the mass of the mammal measured in kg.

Figure 2 illustrates the linear relationship between $\log_{10} h$ and $\log_{10} m$

The line meets the vertical $\log_{10} h$ axis at 2.25 and has a gradient of $-0.235$

(a) Find, to 3 significant figures, the value of $p$ and the value of $q$.

**(3)**

A particular mammal has a mass of 5 kg and a resting heart rate of 119 beats per minute.

(b) Comment on the suitability of the model for this mammal.

**(3)**

(c) With reference to the model, interpret the value of the constant $p$.

**(1)**

a) As the relationship between $\log_{10} h$ and $\log_{10} m$ is linear, we can write it as $y = mx + c$

where $x = \log_{10} m$, $y = \log_{10} h$, $m = -0.235$, $c = 2.25$

$\Rightarrow \log_{10} h = -0.235 \log_{10} m + 2.25$

$h = 10^{-0.235 \log_{10} m} \times 10^{2.25}$

$h = m^{-0.235} \times 10^{2.25}$ (1) $= pm^q$

$p = 10^{2.25}$ (1) $= 178$ and $q = -0.235$ (1)

b) If $m = 5$ kg, then the model predicts

$$h = 178 \times 5^{-0.235} = 122 \text{ beats per minute}$$ (1) (1)

This is accurate to the measured heart rate within 2 significant figures. So, the model is suitable. (1)

c) $p$ would be the resting heart rate in bpm of a mammal with a mass of 1 kg. (1)

**2.** The distance a particular car can travel in a journey starting with a full tank of fuel was investigated.

- From a full tank of fuel, 40 litres remained in the car's fuel tank after the car had travelled 80 km
- From a full tank of fuel, 25 litres remained in the car's fuel tank after the car had travelled 200 km

Using a **linear model**, with $V$ litres being the volume of fuel remaining in the car's fuel tank and $d$ km being the distance the car had travelled,

(a) find an equation linking $V$ with $d$.

**(4)**

Given that, on a particular journey

- the fuel tank of the car was initially full
- the car continued until it ran out of fuel

find, according to the model,

(b) (i) the initial volume of fuel that was in the fuel tank of the car,

(ii) the distance that the car travelled on this journey.

**(3)**

In fact the car travelled 320 km on this journey.

(c) Evaluate the model in light of this information.

**(1)**

a) for equation of linear model V and d,

$V = ad + b$ (1), where a and b are unknown.

After 80 km journey :

$40 = 80a + b$ — ① (1)

After 200 km journey :

$25 = 200a + b$ — ② (1)

substitute ② into ①

$40 = 80a + (25 - 200a)$

$120a = -15$

$a = -\frac{1}{8}$ , $b = 50$

$\therefore V = -\frac{1}{8}d + 50$ (1)

(b) (i) initial volume of the fuel is when $d = 0$.

$V = -\frac{1}{8}(0) + 50$

$= 50$ litres (1)

(ii) Total distance travelled is when $V = 0$

$0 = -\frac{1}{8}d + 50$

$d = 50 \times 8$

$= 400$ km (1)

(c) when $d = 320$ km,

$V = -\frac{1}{8}(320) + 50$

$= 10$ litres

$\therefore$ Concludes that this is a poor model because 10 litres of fuel is significantly more than empty tank. (1)

**3.** The height, $h$ metres, of a plant, $t$ years after it was first measured, is modelled by the equation

$$h = 2.3 - 1.7e^{-0.2t} \qquad t \in \mathbb{R} \quad t \geqslant 0$$

Using the model,

(a) find the height of the plant when it was first measured,

**(2)**

(b) show that, exactly 4 years after it was first measured, the plant was growing at approximately 15.3 cm per year.

**(3)**

According to the model, there is a limit to the height to which this plant can grow.

(c) Deduce the value of this limit.

**(1)**

a) t years = years after first measured

first measured = 0 years

so, height when plant was first measured :

$h = 2.3 - 1.7e^{-0.2(0)}$ (1)

$= 2.3 - 1.7 = 0.6$ m (1)

calculating rate of change of height of the plant

b) $\frac{dh}{dt} = 0.34e^{-0.2t}$ (1)

when $t = 4$,

$\frac{dh}{dt} = 0.34e^{-0.2(4)}$ (1)

$= 0.153$ m $\cong 15.3$ cm (1)

c) when t approaching $\infty$, the height will be 2.3 m. (1)

2.3 m is the value of the limit.

4. The height, $h$ metres, of a tree, $t$ years after being planted, is modelled by the equation

$$h^2 = at + b \qquad 0 \leqslant t < 25$$

where $a$ and $b$ are constants.

Given that

- the height of the tree was 2.60 m, exactly 2 years after being planted
- the height of the tree was 5.10 m, exactly 10 years after being planted

(a) find a complete equation for the model, giving the values of $a$ and $b$ to 3 significant figures.
**(4)**

Given that the height of the tree was 7 m, exactly 20 years after being planted

(b) evaluate the model, giving reasons for your answer.
**(2)**

(a) $h^2 = at + b$

① : $h = 2.6, \ t = 2$ ← from first bullet point

$2.6^2 = 2a + b$ ①

② : $h = 5.1, \ t = 10$ ← from second bullet point

$5.1^2 = 10a + b$

① : $6.76 = 2a + b$

② : $26.01 = 10a + b$ ① solve simultaneously

② − ① : $19.25 = 8a$

$2.40625 = a$

$6.76 = 2 \times 2.40625$

$6.76 = 4.8125 + b$

$1.9475 = b$

$a = 2.41$ ①

$b = 1.95$ rounded to 2.d.p

$\therefore h^2 = 2.41t + 1.95$ ①

b) Height of the tree when $t = 20$ years :

$h^2 = 2.41(20) + 1.95$

$h^2 = 48.2 + 1.95$

$h^2 = 50.15$

$h = \sqrt{50.15}$

1

$= 7.08$ m $\therefore$ the model is good as 7.08 m is close to 7 m.

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